



AM 117

$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = x - \operatorname{tg} x$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{f(\operatorname{tg} x)}{x^3} &= \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \operatorname{tg}(\operatorname{tg} x)}{x^3} = \lim_{x \rightarrow 0} \frac{\frac{1}{\cos^2 x} - \frac{1}{\cos^2(\operatorname{tg} x)} \cdot \frac{1}{\cos^2 x}}{3x^2} = \lim_{x \rightarrow 0} \frac{1 - \frac{1}{\cos^2(\operatorname{tg} x)}}{3x^2} = \\ &= \frac{1}{3} \lim_{x \rightarrow 0} \frac{\cos^2(\operatorname{tg} x) - 1}{x^2} = -\frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin^2(\operatorname{tg} x)}{x^2} = -\frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin^2(\operatorname{tg} x)}{\operatorname{tg}^2 x} \cdot \frac{\operatorname{tg}^2 x}{x^2} = -\frac{1}{3} \end{aligned}$$

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